Strengthening Convex Relaxations with Bound Tightening for Power Network Optimization

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What do you do after formulating your optimization problem and realizing it is NP-Hard? You can give up your soul to the lord of meta-heuristics, or, you can choose the tumultuous road to convexification. Convex relaxations play a central role in providing optimality guarantees to difficult non-convex problems. These relaxations can also help guide the search for a global optimum.

In our 2015 paper "Strengthening convex relaxations with bound tightening for power network optimization" we address the convexification of the Alternating Current Optimal Power Flow (ACOPF), a fundamental problem in power systems optimization. Our contributions consist in bringing together multiple approaches to try and close the optimality gap on open instances on state-of-the-art benchmarks. Our main goal was to push the limits of techniques that do not require the introduction of binary variables to maintain computational scalability. One of the surprising findings of this paper was the power of bound tightening on top of convex relaxations for power systems: The tightening was able to identify many of the power flow directions on highly complex test cases.

Since the publication of our paper, we continued our efforts in that direction, trying to combine and extend existing methods that do not require domain partitioning for continuous variables. We recently had a breakthrough. Our efforts paid off when we combined determinant cuts [1] capturing the strength of semidefinite programming relaxations with RLT [2] along with optimality-based bound tightening [3], this combination was sufficient to close 98% of open instances without having to branch once. The parallelizable nature of OBBT makes it an ideal candidate for tackling real-world large-scale problems. We are now at the stage of trying to explain why this combination works well and whether it can be successful in other application areas and if it extends to mixed-integer settings.
An open source implementation of our contributions can be accessed here and here for the initial ACOPF specific application and here for the general determinant cuts based OBBT algorithm.

[1] H. Hijazi, C. Coffrin, and P. Van Hentenryck, "Polynomial SDP cuts for optimal power flow", 2016 Power Systems Computation Conference (PSCC), pp. 1–7, June 2016.

[2] D. Sherali and A. Alameddine, "A new reformulation-linearization technique for bilinear programming problems", Journal of Global Optimization, vol. 2, no. 4, pp. 379–410, Dec 1992.

[3] Quesada, I., Grossmann, I.E.: "A global optimization algorithm for linear fractional and bilinear programs". Journal of Global Optimization 6, 39–76 (1995).