

Learning the Parameters of Global Constraints for Medical Scheduling

Émilie Picard-Cantin, Mathieu Bouchard,
Claude-Guy Quimper, and Jason Sweeney





Émilie Picard-Cantin



Medical Schedule

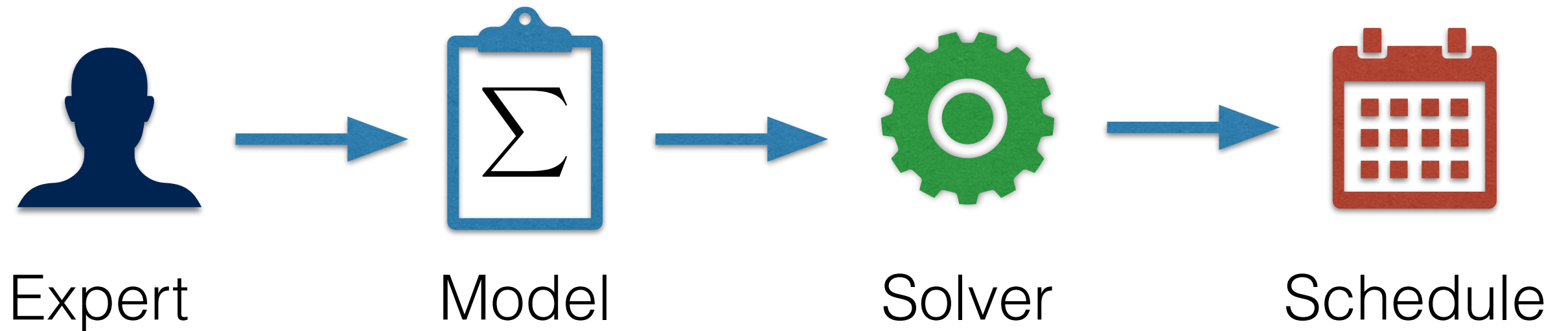
People oriented

	1	2	3	4	5	6	7
							
							
							

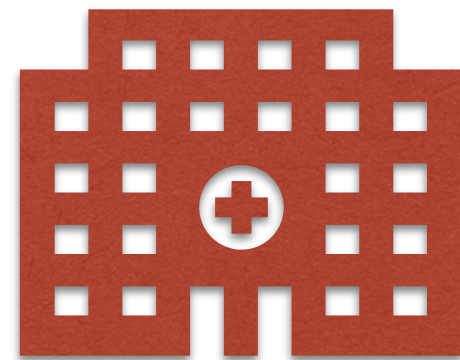
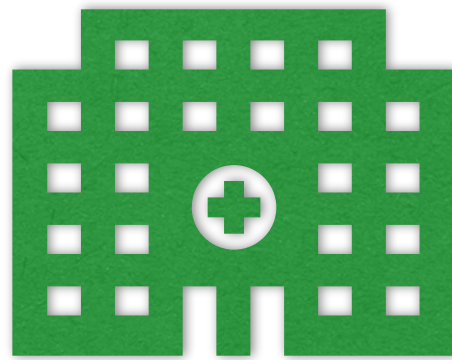
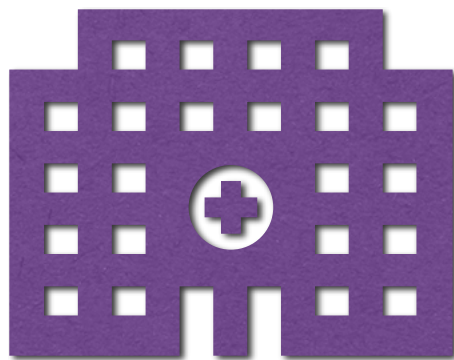
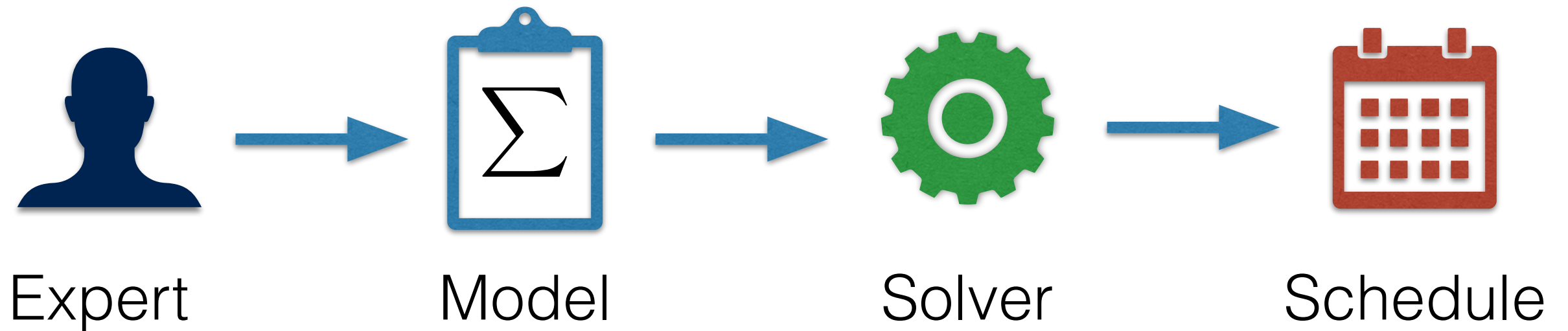
Shift / Task oriented

	1	2	3	4	5	6	7
							
							

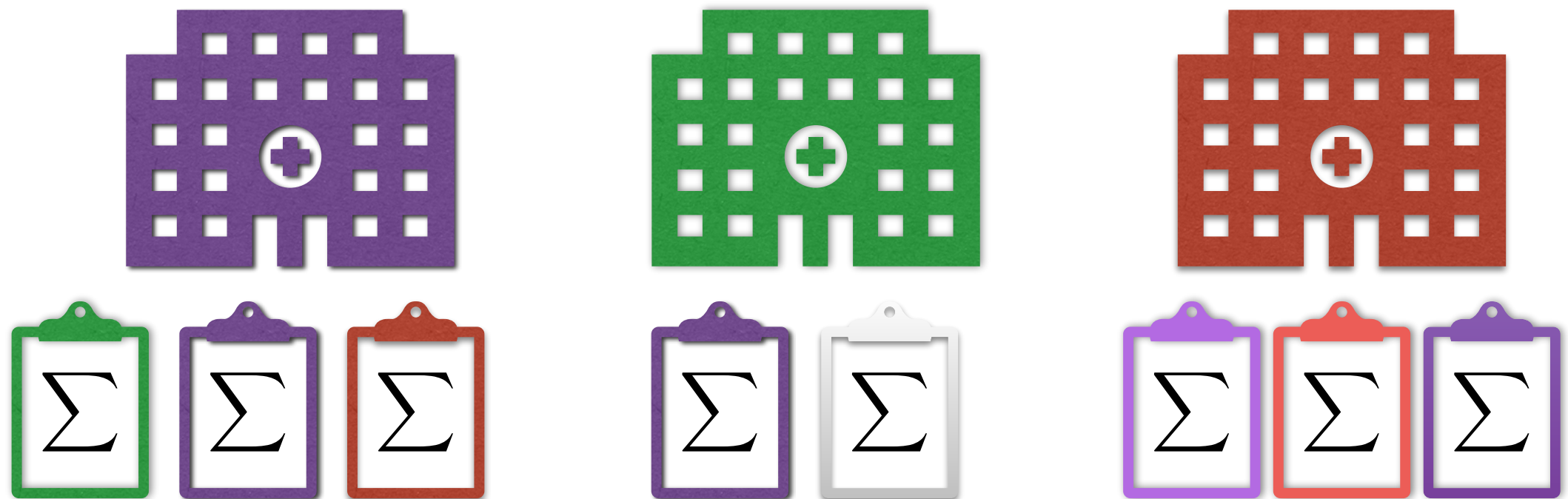
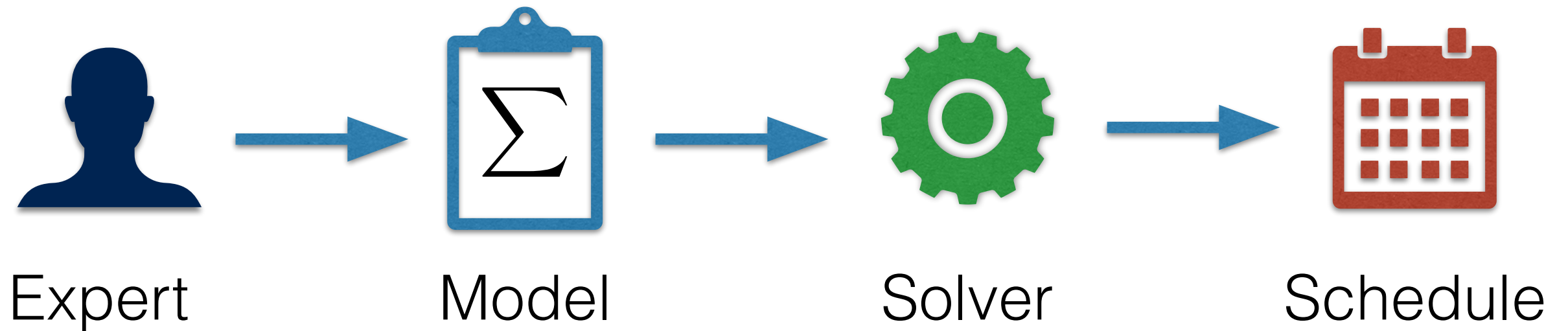
Scheduling Process



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Speeding up the modelling process



Speeding up the modelling process

- **Challenge:** Models differ from one medical team to another

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- **Observation:** There are few differences between each model.

Speeding up the modelling process

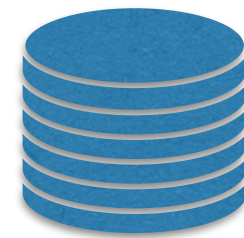
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Speeding up the modelling process

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- **Observation:** There are few differences between each model.
- **Opportunity:** For legal reasons, hospitals keep a history of their schedules.
- **Goal:** To learn the models from historical data.

Recommender System

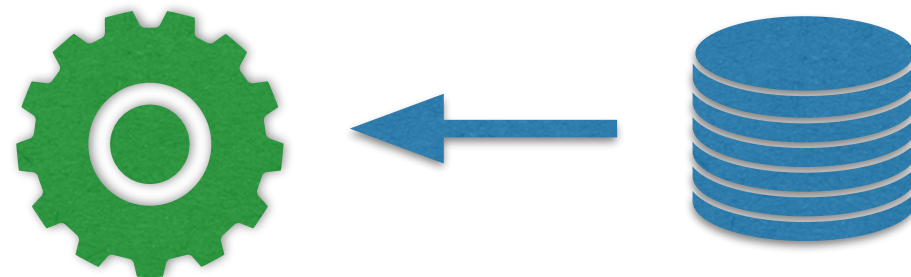
Past
schedules



Recommender System

Recommender
System

Past
schedules

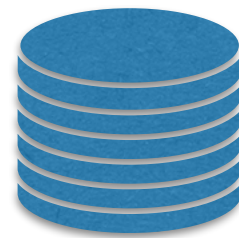


Recommender System

Discovered
Constraints

Recommender
System

Past
schedules

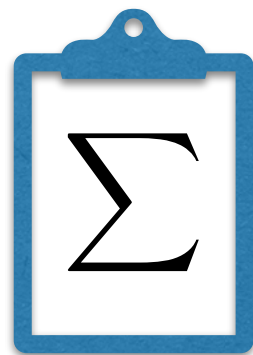
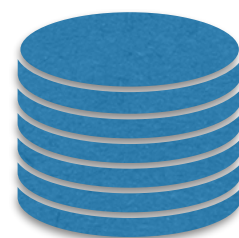


Recommender System

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Past
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Expert

Model

Solver

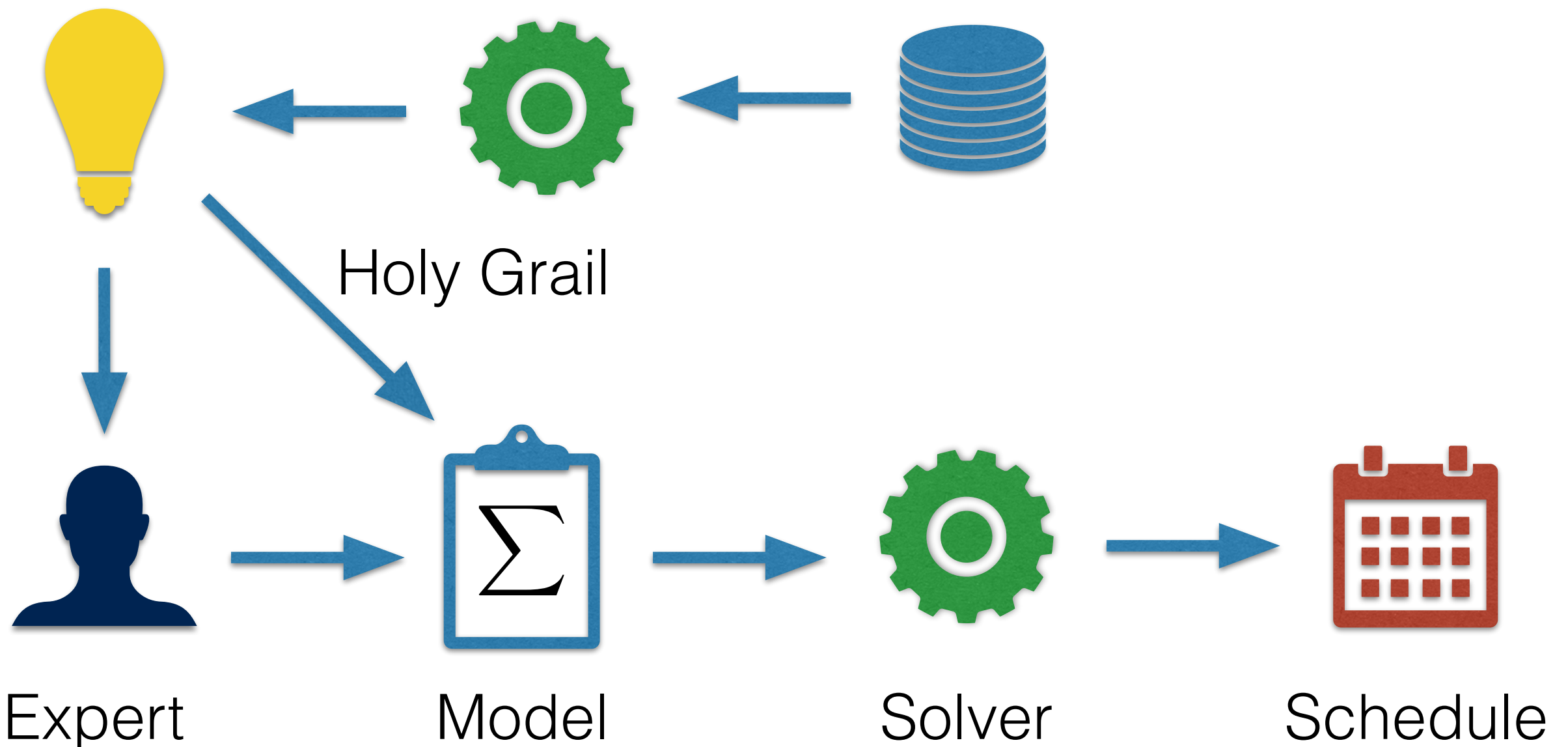
Schedule

Recommender System

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Past
schedules

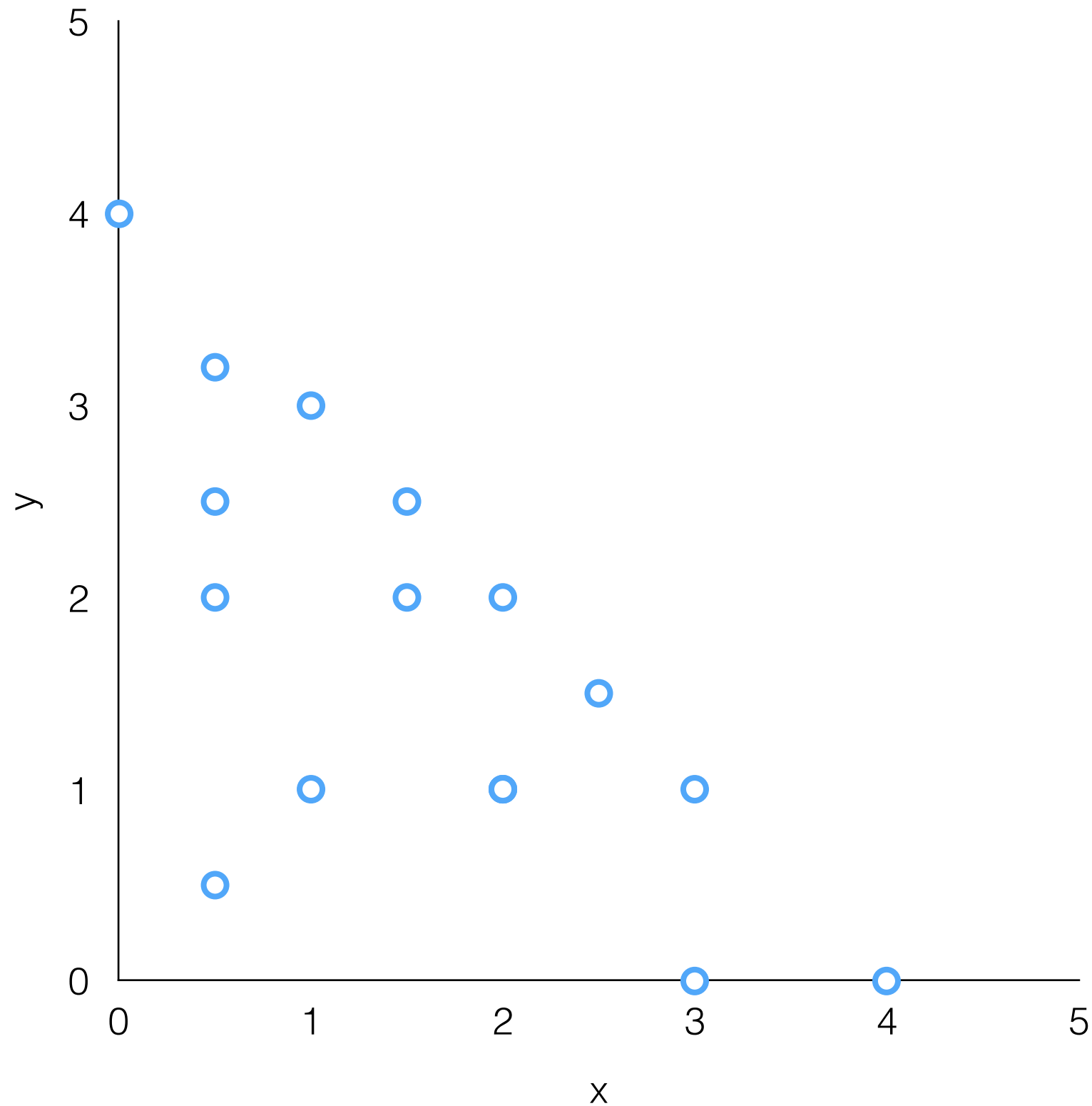


How to learn a constraint?

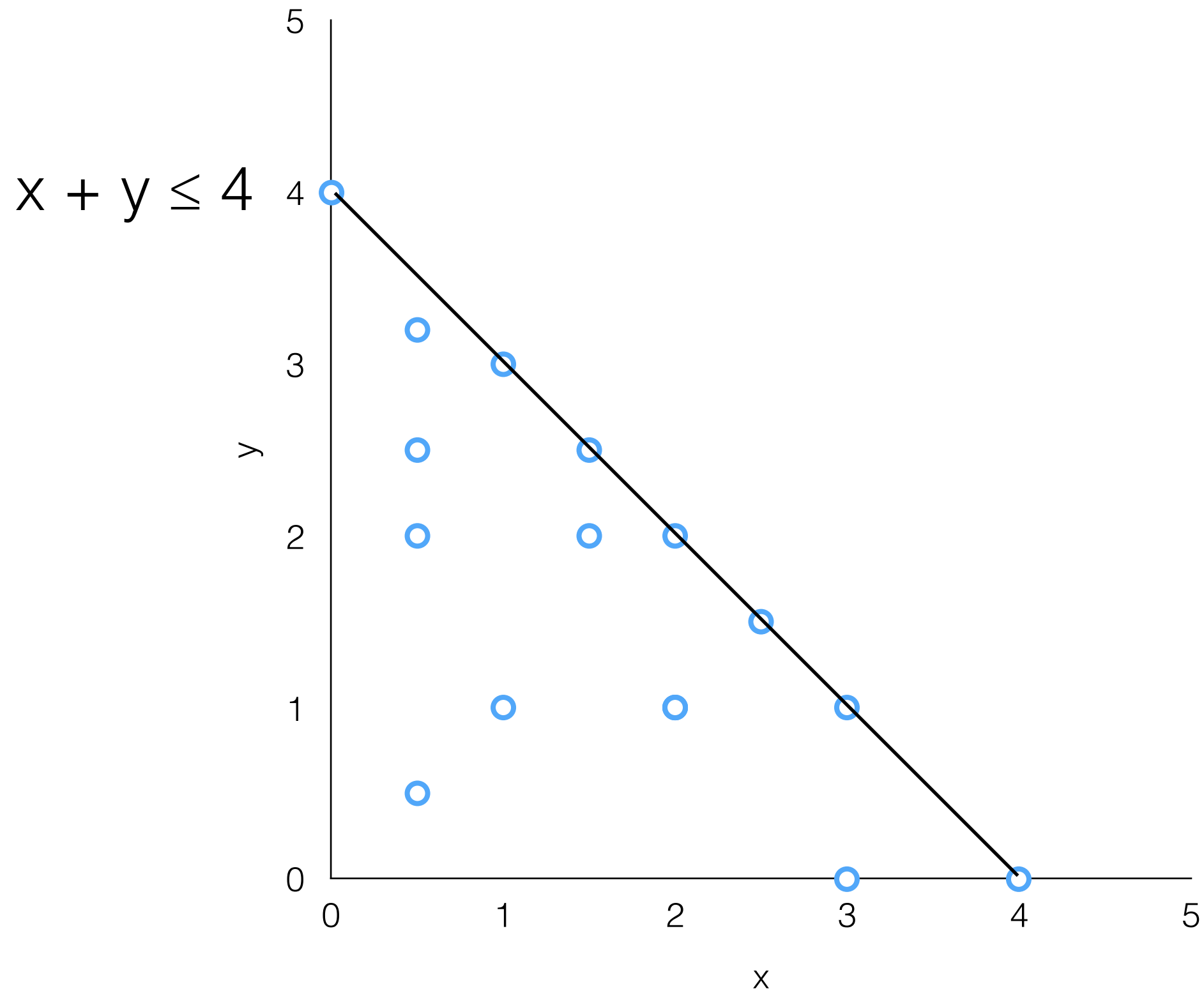
1	2	3	4	5	6	7
						

- Do we have a limit of:
 - 2 night-shifts per week?
 - 1 night-shift every 3 days?

Which constraint was imposed?



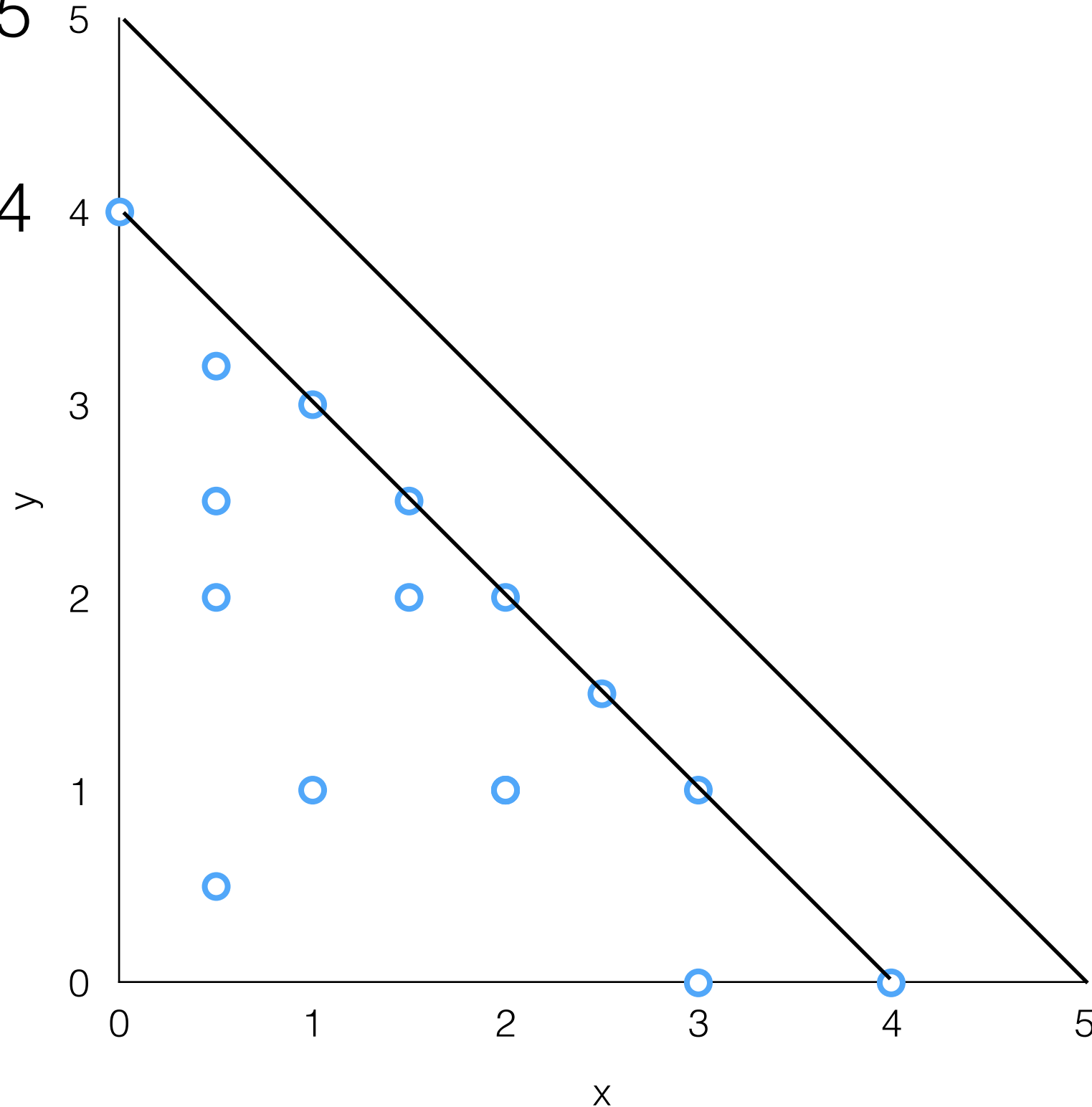
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Which constraint was imposed?

$$x + y \leq 5$$

$$x + y \leq 4$$



Problem Definition

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$$G_C(\vec{\alpha}) = \sum_{\vec{X} | C(\vec{X}, \vec{\alpha})} P(\vec{X})$$

- Finding $\vec{\alpha}$ consists in solving:

$$\min_{\vec{\alpha}} G_C(\vec{\alpha})$$

$$C(\vec{X}, \vec{\alpha}) \quad \forall \vec{X} \in \text{Examples}$$

How to compute $G_C(\vec{a})$?

- Enumerating and summing the probability of all solutions of a constraint is slow.
- We mainly developed two techniques to compute or bound this probability
 - Using Markov chains
 - Using dynamic programming

Markov Chains

- Some constraints can naturally be encoded with an automaton.

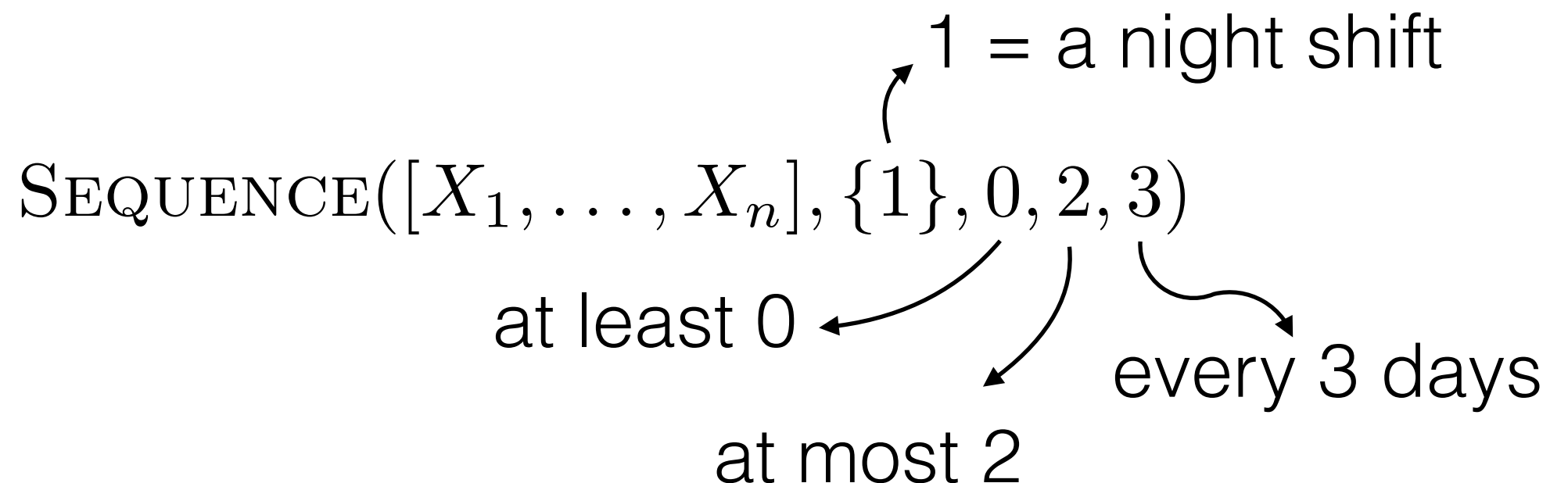
$\text{SEQUENCE}([X_1, \dots, X_n], \{1\}, 0, 2, 3)$

1 = a night shift

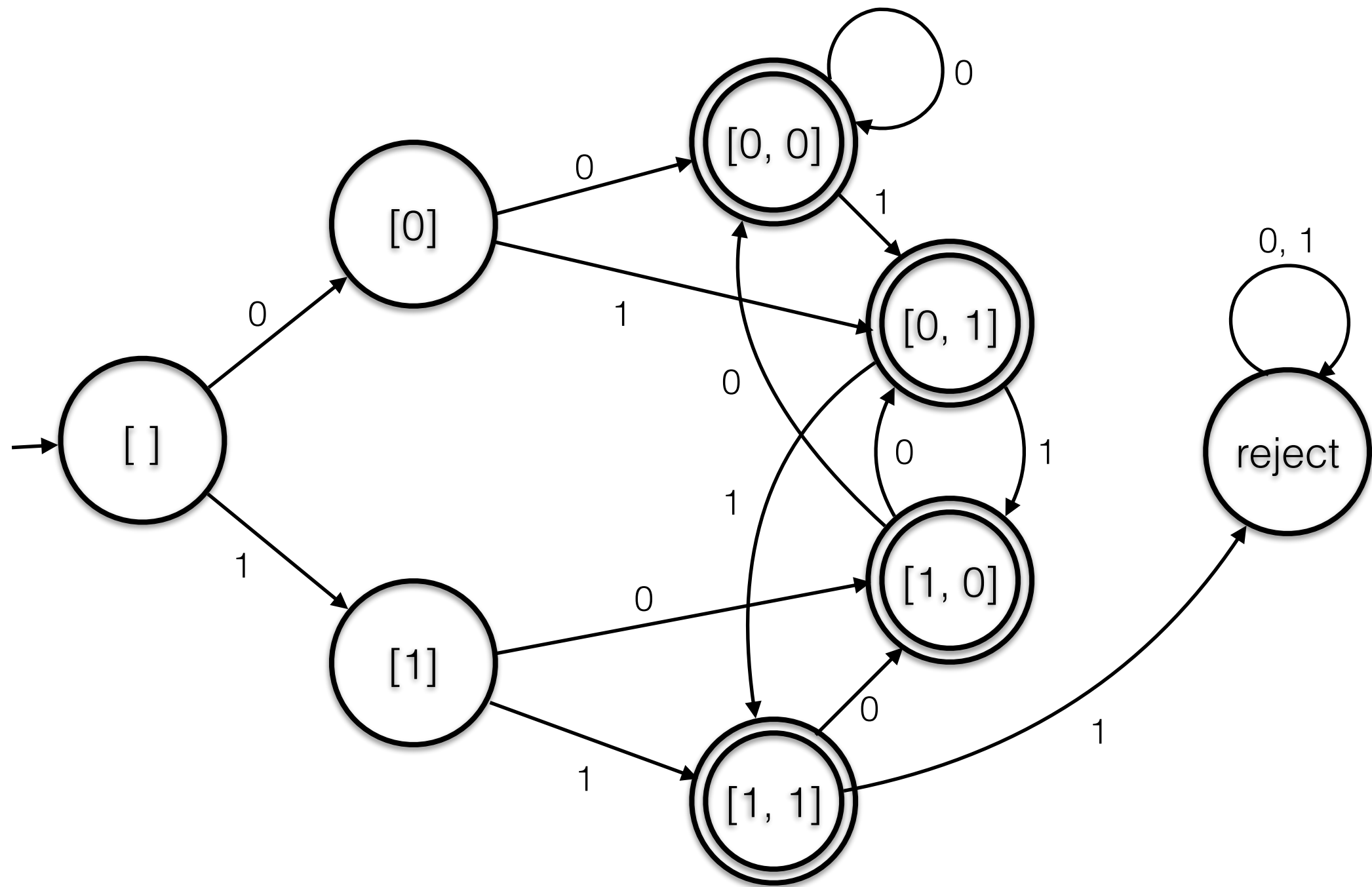
at least 0

at most 2

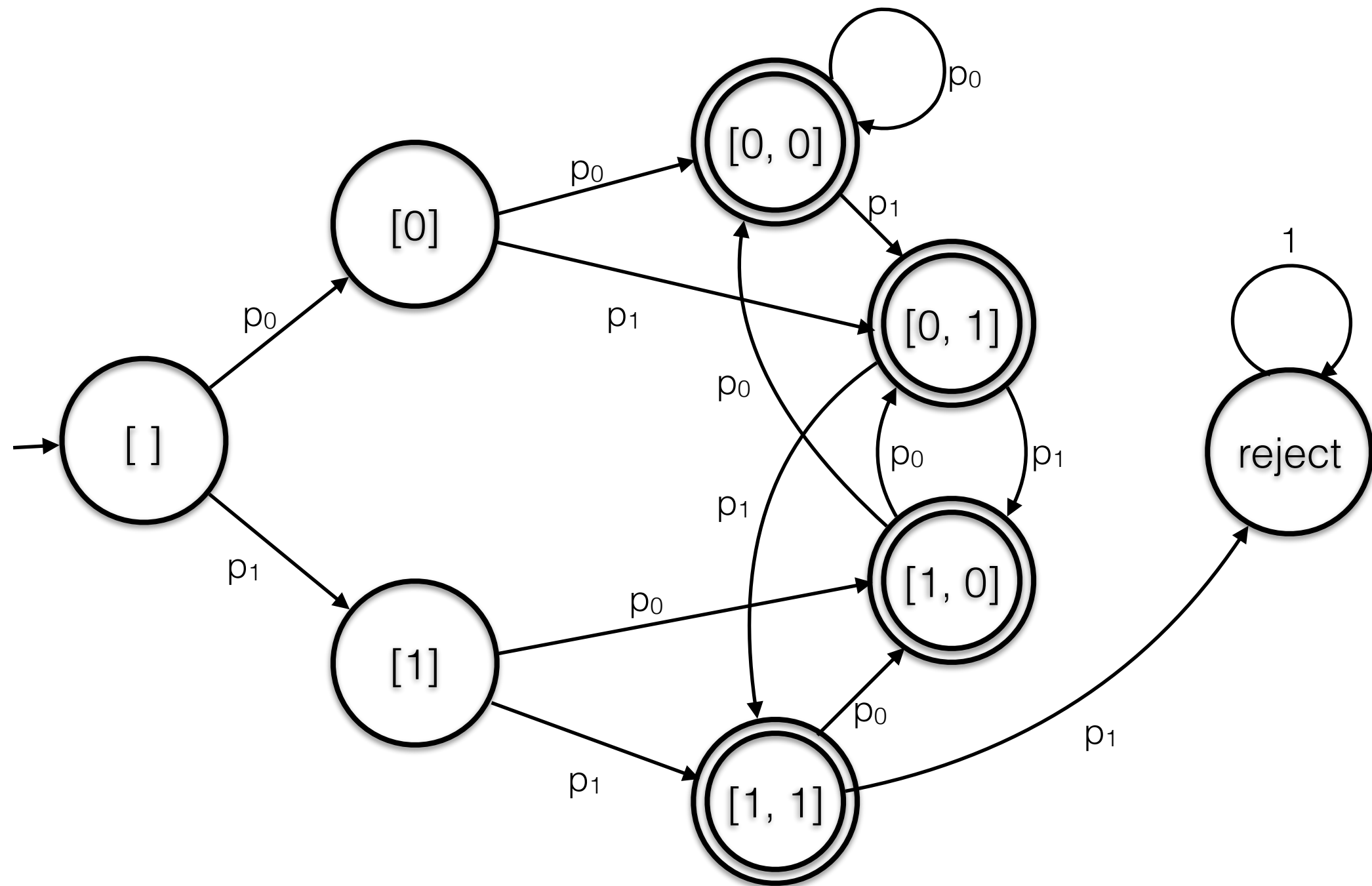
every 3 days



Sequence Automaton



Sequence Markov Chain



Computing $G_{\text{SEQUENCE}}(\vec{\alpha})$

- Let $M_{\vec{\alpha}}$ be the transition matrix of the Markov chain for the constraint with parameters $\vec{\alpha}$.
- One can compute the probability of reaching the reject state after reading n characters by computing $M_{\vec{\alpha}}^n$.
- For every combination of $\vec{\alpha}$, compute $M_{\vec{\alpha}}^n$ and evaluate $G_C(\vec{\alpha})$.
- Keep $\vec{\alpha}$ that minimizes $G_C(\vec{\alpha})$.

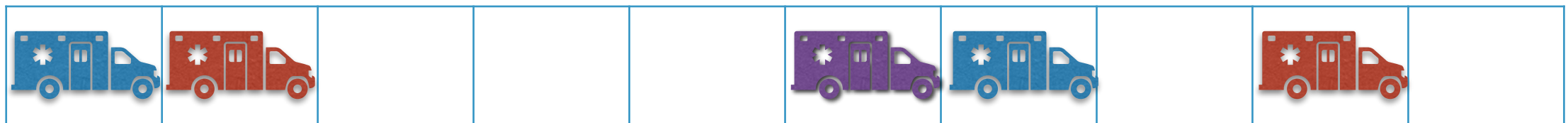
When parameters are sets

- If the parameter contains a set, there is an exponential number of combinations to explore.

AMONG($[X_1, \dots, X_n], l, u, \vec{z}$)

SEQUENCE($[X_1, \dots, X_n], l, u, w, \vec{z}$)

SUBSETFOCUS($[X_1, \dots, X_n], l, m, \vec{z}$)



Branch & Bound

$$\min_{\vec{\alpha}} G_C(\vec{\alpha})$$

$$C(\vec{X}, \vec{\alpha}) \quad \forall \vec{X} \in \text{Examples}$$

- We use this strategy to compute a bound on $G_C(\vec{\alpha})$

$$p = \sum_v z_v \cdot p_v$$

$$G_C(\vec{\alpha}) \geq G_C(\text{ext}(\vec{\alpha}))$$

$$\geq \sum_{k=0}^n D[k, \text{ext}(\vec{\alpha})] \left(\min_p p^k (1-p)^{n-k} \right)$$

Branch & Bound

$$\min_{\vec{\alpha}} G_C(\vec{\alpha})$$

$$\forall \vec{X} \in \text{Examples}$$

Probability that a value
belongs to the set.

- We use this to compute a bound on $G_C(\vec{\alpha})$

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Branch & Bound

$$\min_{\vec{\alpha}} G_C(\vec{\alpha})$$

$$C(\vec{X}, \vec{\alpha})$$

Examples

- We use this strategy to find a lower bound on $G_C(\vec{\alpha})$

Set parameters to extreme values (requires monotonicity)

$$p = \sum_v z_v \cdot p_v$$

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- We use this strategy to compute a bound on $G_C(\vec{\alpha})$

Count how many solutions exist with exactly k values in the set

$$G_C(\vec{\alpha}) \geq \sum_{k=0}^n D[k, \text{ext}(\vec{\alpha})] p_v^k (1 - p_v)^{n-k}$$

$$\geq \sum_{k=0}^n D[k, \text{ext}(\vec{\alpha})] \left(\min_p p^k (1 - p)^{n-k} \right)$$

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Count how many solutions exist with exactly k values in the set

Probability of a random assignment with k values in the set

$$G_C(\vec{\alpha}) \geq \sum_{k=0}^n D[k, ext(\vec{\alpha})] \left(\min_p p^k (1-p)^{n-k} \right)$$

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Studied Constraints

SUBSETFOCUS

SEQUENCE

AMONG

GCC

ATMOSTNVALUE

ATLEASTNVALUE

ATMOSTBALANCE

ATLEASTBALANCE

Experiments

Num. of examples	Rank of initial constraint				Num. of instances
	1	2	3	∞	
1	8	46	1	545	600
2	42	119	0	439	600
3	78	148	0	374	600
4	105	172	0	323	600
5	139	170	0	291	600
10	261	117	0	222	600

Table 1: Results for SUBSETFOCUS. Number of instances for which the initial constraint was ranked first, second, third or was not found.

Conclusion

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- The system is not used!
- It could have saved hundreds of hours in expert time.